

BRIGHT SCHOLARS

MATH SERIES • BOOK 3

MASTERING
ALGEBRA 2

BOOK 3

Grades 10–11

A Complete Practice Workbook for Functions, Quadratics, Polynomials,
Rational and Radical Expressions, Exponentials, Logarithms, Sequences, and
Modeling

Learn it. Practice it. Prove it.

FUNCTIONS • POLYNOMIALS • RATIONALS • RADICALS • LOGARITHMS • SEQUENCES

THIS BOOK BELONGS TO

Name _____

Class / Period _____

Teacher _____

School Year _____

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Algebra 2 Quick Reference

Keep this page handy to recall a rule or formula. It is not a substitute for showing your reasoning.

Topic	Rule or formula
Lines	Slope $m = \frac{y_2 - y_1}{x_2 - x_1}$. Point-slope: $y - y_1 = m(x - x_1)$. Parallel lines have equal slopes; perpendicular slopes multiply to -1 .
Quadratic forms	Standard $y = ax^2 + bx + c$, vertex at $x = \frac{-b}{2a}$. Vertex $y = a(x - h)^2 + k$, vertex (h, k) . Factored $y = a(x - r)(x - s)$, zeros r and s .
Quadratic formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Discriminant $b^2 - 4ac$: positive, two real; zero, one real; negative, two nonreal.
Complex numbers	$i = \sqrt{-1}$, $i^2 = -1$. $(a + bi)(a - bi) = a^2 + b^2$.
Polynomials	Remainder theorem: the remainder of $p(x) \div (x - c)$ is $p(c)$. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$; $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.
Rational expressions	Factor first, state restrictions from the original denominators, then cancel common factors. Check every solution against the restrictions.
Radicals	$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$. $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$. $\sqrt{x^2} = x $. Isolate the radical, raise both sides to the power, and check for extraneous solutions.
Exponential models	$y = a(1 + r)^t$ (growth), $y = a(1 - r)^t$ (decay), $A = P(1 + \frac{r}{n})^{nt}$, $A = Pe^{rt}$.
Logarithms	$\log_b x = y$ means $b^y = x$. $\log_b(MN) = \log_b M + \log_b N$; $\log_b \frac{M}{N} = \log_b M - \log_b N$; $\log_b M^p = p \log_b M$; $\log_b x = \frac{\ln x}{\ln b}$.
Sequences and series	Arithmetic: $a_n = a_1 + (n - 1)d$, $S_n = \frac{n(a_1 + a_n)}{2}$. Geometric: $a_n = a_1 r^{n-1}$, $S_n = \frac{a_1(1 - r^n)}{1 - r}$. Infinite ($ r < 1$): $S = \frac{a_1}{1 - r}$.
Transformations	$y = a f(b(x - h)) + k$: shift right h and up k ; $a < 0$ reflects over the x -axis; $ a > 1$ stretches vertically.
Inverses	Swap x and y , then solve for y . Check that $f(f^{-1}(x)) = x$. The graphs are reflections over $y = x$.

Quadratic Functions and Equations

? BIG QUESTION

A thrown ball, a bridge cable, and a satellite dish all trace the same kind of curve. How can one equation tell you exactly where that curve turns and where it meets the ground?

BY THE END OF THIS UNIT, I CAN

- ☐ move among the standard, vertex, and factored forms of a quadratic.
- ☐ graph a parabola from its vertex, axis of symmetry, and intercepts.
- ☐ solve quadratic equations by factoring, the quadratic formula, and completing the square.
- ☐ use the discriminant and complex numbers to describe all the roots of a quadratic.

UNIT ROADMAP

- 3.1 What Is a Quadratic? p. 54
- 3.2 Vertex from Standard Form p. 55
- 3.3 Vertex Form p. 56
- 3.4 Factored Form and Solving by Factoring p. 58
- 3.5 The Quadratic Formula p. 59
- 3.6 Completing the Square p. 61
- 3.7 Completing the Square When $a \neq 1$ p. 62
- 3.8 Complex Numbers and Nonreal Quadratic Solutions p. 63

Suggested time: 3 weeks

Standards: A-SSE.B.3a; A-SSE.B.3b; A-REI.B.4a; A-REI.B.4b; F-IF.C.7a; F-IF.C.8a; N-CN.A.1; N-CN.A.2; N-CN.C.7

KEY VOCABULARY

quadratic function parabola vertex

axis of symmetry zeros discriminant

standard form vertex form factored form

completing the square imaginary unit

complex number

Vertex from Standard Form

KEY TERMS vertex • axis of symmetry • y-intercept • reflection

In standard form, the vertex is not visible, but its x-coordinate comes from a short formula. Substitute that x-value back into the function to get the y-coordinate.

$$x = \frac{-b}{2a} \quad \text{then} \quad y = f\left(\frac{-b}{2a}\right)$$

EXAMPLE 1 Find the vertex of $f(x) = 2x^2 - 8x + 3$

STEP 1 Identify the coefficients: $a = 2$, $b = -8$, $c = 3$.

STEP 2 $x = \frac{-b}{2a} = \frac{-(-8)}{2(2)} = \frac{8}{4} = 2.$

STEP 3 $y = f(2) = 2(4) - 16 + 3 = -5.$

ANSWER Vertex $(2, -5)$. Since $a = 2 > 0$, it is a minimum.

STEPS TO GRAPH FROM STANDARD FORM

1. Find the vertex using $x = \frac{-b}{2a}$, and draw the axis of symmetry through it.
2. Plot the y-intercept $(0, c)$.
3. Reflect the y-intercept across the axis of symmetry to get a second point at the same height.
4. If you can, factor or use the quadratic formula to find the x-intercepts.
5. Sketch the parabola, opening up if $a > 0$ or down if $a < 0$.

EXAMPLE 2 Graph $f(x) = x^2 - 2x - 3$

STEP 1 Vertex: $x = \frac{-(-2)}{2(1)} = 1$ and $f(1) = 1 - 2 - 3 = -4$, so the vertex is $(1, -4)$. The axis is $x = 1$.

STEP 2 y-intercept: $(0, -3)$.

STEP 3 $(0, -3)$ is 1 unit left of the axis, so its mirror point is 1 unit right: $(2, -3)$.

STEP 4 Factor: $(x - 3)(x + 1) = 0$, so the x-intercepts are $(3, 0)$ and $(-1, 0)$.

ANSWER Plot the five points and draw a smooth U-shaped curve opening up (Figure 3.1).

 EXAMPLE 1 Evaluate $f(x) = 3 \cdot 2^x$ at $x = 0, 1, 2$, and -1

STEP 1 $f(0) = 3 \cdot 2^0 = 3 \cdot 1 = 3$.

STEP 2 $f(1) = 3 \cdot 2 = 6$, and $f(2) = 3 \cdot 4 = 12$.

STEP 3 A negative exponent means a reciprocal: $f(-1) = 3 \cdot 2^{-1} = 3 \cdot \frac{1}{2} = \frac{3}{2}$.

ANSWER $f(0) = 3, f(1) = 6, f(2) = 12, f(-1) = \frac{3}{2}$

 EXAMPLE 2 Describe $h(x) = 40(0.8)^x$

STEP 1 The base is $b = 0.8$, and $0 < 0.8 < 1$, so h shows decay.

STEP 2 The initial value is $a = 40$, so the y -intercept is $(0, 40)$.

STEP 3 The horizontal asymptote is $y = 0$. Because $a > 0$, every output is positive.

ANSWER Decay; y -intercept $(0, 40)$; asymptote $y = 0$; domain $(-\infty, \infty)$; range $(0, \infty)$

 QUICK CHECK

1 $5^0 = \underline{\hspace{1cm}}$ $2^{-3} = \underline{\hspace{1cm}}$ $3^{1/2} \approx \underline{\hspace{1cm}}$ (use a calculator; round to the nearest thousandth)

2 For $f(x) = 4 \cdot 2^x$: $f(0) = \underline{\hspace{1cm}}$, $f(3) = \underline{\hspace{1cm}}$, $f(-2) = \underline{\hspace{1cm}}$

Unit 9 Practice



SET 1: TRANSFORMATIONS

Describe the transformations from $f(x)$ to $g(x)$. State the vertex, starting point, or asymptotes where they apply.

1 $f(x) = x^2$; $g(x) = (x + 3)^2 - 2$

2 $f(x) = \sqrt{x}$; $g(x) = \sqrt{x - 4} + 1$

3 $f(x) = |x|$; $g(x) = 3|x| - 4$

4 $f(x) = x^3$; $g(x) = -(x - 1)^3 + 2$

5 $f(x) = \frac{1}{x}$; $g(x) = \frac{1}{x - 5} + 3$

6 $f(x) = 2^x$; $g(x) = 2^{x+1} - 4$

7 $f(x) = x^2$; $g(x) = -\frac{1}{2}x^2$

8 $f(x) = |x|$; $g(x) = |x - 2| + 6$

Write the equation of $g(x)$.

9 $f(x) = x^2$ is reflected over the x-axis, then shifted right 6 and down 1.

10 $f(x) = \sqrt{x}$ is shifted left 3 and up 4.

11 $f(x) = |x|$ is stretched vertically by a factor of 4 and reflected over the x-axis.

Error Analysis

Common Mistake	Do This Instead
MISTAKE Subtracting only the first term of the second polynomial: $(5x^2 + 2x) - (3x^2 - 4x) = 2x^2 - 2x$.	FIX Distribute the minus sign to every term: $5x^2 + 2x - 3x^2 + 4x = 2x^2 + 6x$.
MISTAKE Squaring a binomial term by term: $(x + 4)^2 = x^2 + 16$.	FIX Write it as $(x + 4)(x + 4)$ and use all four products: $x^2 + 8x + 16$.
MISTAKE Skipping a missing power in synthetic division: using 1, -3, 1 for $x^3 - 3x + 1$.	FIX Hold every place with a coefficient: 1, 0, -3, 1.
MISTAKE Using the number in the divisor as k without changing its sign: $k = 4$ for $x + 4$.	FIX Write the divisor as $x - k$: $x + 4 = x - (-4)$, so $k = -4$.
MISTAKE Mixing up the signs in the cube patterns: $y^3 + 1 = (y + 1)(y^2 + y + 1)$.	FIX Same sign, opposite sign, always positive: $y^3 + 1 = (y + 1)(y^2 - y + 1)$.
MISTAKE Reading end behavior from the first term written: $f(x) = 3x - 2x^5$ has "positive" ends.	FIX Find the highest-degree term, $-2x^5$: odd degree, negative coefficient, so left up and right down.

Each box shows a student's incorrect work. Describe the error in words, then write the correct solution.

! STUDENT WORK Multiply $(x + 2)(x^2 - 2x + 4)$

STEP 1 This matches the sum of cubes pattern $(a + b)(a^2 - ab + b^2)$ with $a = x$ and $b = 2$.

STEP 2 So the product is $a^3 - b^3 = x^3 - 8$.

ANSWER $x^3 - 8$

- 1 Find the sign error in how the student used the pattern. Show the correct work.

Unit 5 Quiz

Time goal: 35 minutes. 9 questions. Show your work for full credit and state all restrictions.

1 Simplify $\frac{x^2-9}{x-3}$.

(A) $x-3$

(B) $\frac{x-3}{x+3}$

(C) $x+3$

(D) -3

2 What are the restrictions on $\frac{x+5}{x^2-4}$?

(A) $x \neq \pm 2$

(B) $x \neq -5$

(C) $x \neq 4$

(D) no restrictions

3 Solve $\frac{6}{x} = 2$.

(A) $x = 12$

(B) $x = 4$

(C) $x = \frac{1}{3}$

(D) $x = 3$

4 What is the LCD of $\frac{3}{x-1}$ and $\frac{5}{x^2-1}$?

(A) $x-1$

(B) x^2-1

(C) $(x-1)^2$

(D) $(x-1)^2(x+1)$

5 Simplify $\frac{x^2+7x+10}{x^2-25}$.

6 Multiply $\frac{x^2-4}{x+3} \cdot \frac{x^2+6x+9}{x-2}$.

Mid-Book Exam

Units 1–5 • Suggested time: 90 minutes

Name _____ Date _____ Score _____

A Multiple Choice	B Mixed Skills	C Short Answer	D Modeling	Total
____ / 40	____ / 18	____ / 24	____ / 18	____ / 100

? EXAM DIRECTIONS

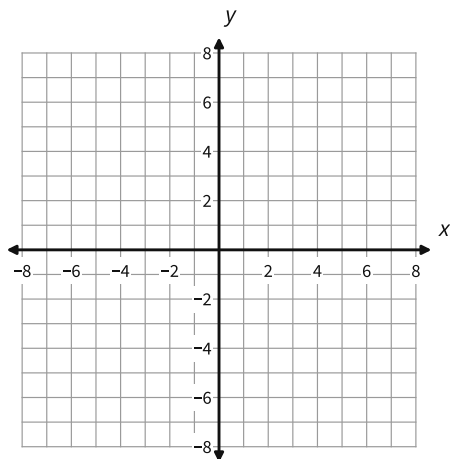
This exam covers Unit 1 (Algebra 1 Review and Foundations), Unit 2 (Linear Functions and Systems), Unit 3 (Quadratic Functions and Equations), Unit 4 (Polynomial Functions and Operations), and Unit 5 (Rational Expressions and Equations). It has 36 questions. Show all work for full credit on every question that is not multiple choice, and state restrictions wherever a variable appears in a denominator.

Section A: Multiple Choice (40 points, 2 points each)

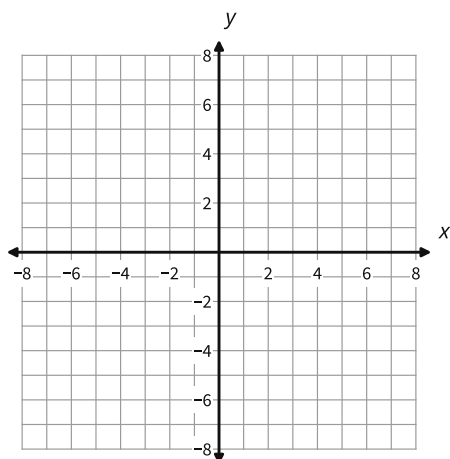
- Solve $4(2x - 3) = 20$.
 (A) $x = 2$ (B) $x = 4$ (C) $x = 8$ (D) $x = -4$
- What is the slope of the line through $(-2, 6)$ and $(2, -2)$?
 (A) 2 (B) $-\frac{1}{2}$ (C) -2 (D) $\frac{1}{2}$
- Factor $x^2 - 121$.
 (A) $(x - 121)(x + 1)$ (B) $(x - 11)^2$
 (C) prime (D) $(x - 11)(x + 11)$
- For $f(x) = 3x^2 - 2x + 4$, find $f(2)$.
 (A) 12 (B) 8 (C) 16 (D) 20
- What is the domain of $f(x) = \sqrt{x - 4}$?
 (A) $x > 4$ (B) $x \neq 4$
 (C) $x \geq 4$ (D) all real numbers
- Solve the system $y = 4x - 3$ and $x + y = 12$.
 (A) $(9, 3)$ (B) $(3, 9)$ (C) $(2, 5)$ (D) $(4, 8)$

Section C: Graphing (25 points, 5 points each)

- 36 Sketch $f(x) = (x + 2)^2 - 3$. State the vertex, the axis of symmetry, and the y-intercept.



- 37 Sketch $g(x) = 2|x + 1| - 3$. State the vertex and the slopes of the two arms.



17. **B.** Odd degree with a negative leading coefficient: rises on the left, falls on the right.
18. **C.** $(2x^3)^2 = 2^2(x^3)^2 = 4x^6$
19. **D.** Parallel lines have equal slopes.
20. **A.** $x - 1 = 3$, so $x = 4$ ($x \neq 1$).

Section B: Mixed Skills (p. 129)

21. $y = 2x + 5$ ($y - 3 = 2(x + 1)$)
22. $(4, -11)$ ($x = -\frac{b}{2a} = \frac{8}{2} = 4$; $f(4) = 16 - 32 + 5 = -11$)
23. $(3x + 4)(x + 2)$ ($ac = 24$: 6 and 4; $3x^2 + 6x + 4x + 8$)
24. 2 (Remainder Theorem: $f(2) = 8 - 8 + 6 - 4 = 2$)
25. $x - 8$; $x \neq -8$ ($\frac{(x-8)(x+8)}{x+8}$)
26. $\frac{5x-1}{(x-1)(x+1)}$; $x \neq \pm 1$ ($2(x+1) + 3(x-1) = 5x-1$)

Section C: Short Answer (p. 130)

27. $x = 17$ ($3x - 10 = 2x + 7$)
28. $(5, 3)$ (add: $3x = 15$, so $x = 5$ and $y = 3$)
29. $x = 3 \pm \sqrt{5}$ ($x = \frac{6 \pm \sqrt{36-16}}{2} = \frac{6 \pm 2\sqrt{5}}{2}$)
30. $f(x) = (x+5)^2 - 7$ ($x^2 + 10x + 25 - 25 + 18$)
31. $(x-1)(x+1)(x^2+1)$ ($(x^2-1)(x^2+1)$; x^2+1 does not factor over the reals)
32. $x = 7$; $x \neq 2$ ($5 = x - 2$)
33. $x \neq 3$ and $x \neq -3$ ($x^2 - 9 = (x-3)(x+3)$)
34. Zeros $x = 2$ and $x = 5$ ($(x-2)(x-5)$); y -intercept $(0, 10)$

Section D: Modeling (p. 131)

35. (a) $A(m) = 18 + 0.04m$; $B(m) = 34$. (b) $18 + 0.04m = 34$ gives $0.04m = 16$, so $m = 400$ minutes. (c) Plan B: $A(500) = 18 + 20 = \$38$ versus $\$34$, so Plan B is cheaper by $\$4$.
36. (a) $V(x) = x(10 - 2x)(14 - 2x)$. (b) $0 < x < 5$ (the 10 in. side needs $10 - 2x > 0$). (c) $V(2) = 2(6)(10) = 120$ cubic inches.

Unit 6: Radical Expressions and Equations

Lesson 6.1 Roots and Rational Exponents (p. 133)

1. 12; 5; -4
2. 9; $\frac{1}{2}$; 10. $(27^{2/3}) = (\sqrt[3]{27})^2 = 3^2$; $16^{-1/4} = \frac{1}{\sqrt[4]{16}} = \frac{1}{2}$

Lesson 6.2 Simplifying Radical Expressions (p. 135)

1. $4\sqrt{3}$ ($48 = 16 \times 3$)
2. $3|x|\sqrt{5}$
3. $2\sqrt[3]{5}$ ($40 = 8 \times 5$)

Lesson 6.3 Operations with Radicals (p. 137)

1. $5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}$
2. $1(5 - 4)$
3. $\frac{12 - 4\sqrt{2}}{7}$. Multiply by $\frac{3 - \sqrt{2}}{3 - \sqrt{2}}$: the denominator is $9 - 2 = 7$.

Lesson 6.4 Solving Radical Equations (p. 138)

1. $x - 4 = 36$, so $x = 40$. Check: $\sqrt{40 - 4} = \sqrt{36} = 6$. True.
2. $x = 4$. Squaring: $2x + 1 = x^2 - 2x + 1$, so $x^2 - 4x = 0$ and $x = 0$ or $x = 4$. Check $x = 4$: $\sqrt{9} = 3 = 4 - 1$. True. Check $x = 0$: $\sqrt{1} = 1$, but $0 - 1 = -1$, so $x = 0$ is extraneous.

Unit 6 Practice (p. 141)

GUIDED PRACTICE

1. $11\sqrt{3}$
2. $\frac{6\sqrt{7}}{7}$

SET 1: SIMPLIFYING

1. $4\sqrt{2}$
2. $7\sqrt{2}$
3. $10\sqrt{2}$
4. $5\sqrt{3}$
5. $6x^2\sqrt{2}$
6. $5x|y|\sqrt{2x}$. ($x \geq 0$ for x^3 to have a real square root, so x needs no bars.)
7. $2x^2|y|\sqrt{2xy}$. (x and y could both be negative, so y needs bars.)
8. $3\sqrt[3]{2}$
9. $3x\sqrt[3]{3x}$
10. $-2y^2\sqrt[3]{2}$
11. $3|a|\sqrt{5b}$
12. $4|x^3y|\sqrt{3xy}$
13. $\sqrt{x}$
14. $\sqrt[4]{x^3}$, or $(\sqrt[4]{x})^3$
15. $5\sqrt[3]{x^2}$. The exponent applies only to x , so the 5 stays outside the radical.
16. $y^{1/5}$
17. $x^{2/3}$
18. $x^{4/3}$

SET 2: OPERATIONS

1. $8\sqrt{2}$
2. $5\sqrt{3}$
3. $5\sqrt{3}(3\sqrt{3} + 2\sqrt{3})$
4. $\sqrt{2}(5\sqrt{2} - 4\sqrt{2})$
5. $16\sqrt{5}(6\sqrt{5} + 10\sqrt{5})$

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Certificate of Completion

This certifies that

has successfully completed

Mastering Algebra 2

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and has shown the persistence, precision, and careful reasoning of a
real mathematician.

TEACHER / PARENT

DATE