

BRIGHT SCHOLARS

MATH SERIES • BOOK 4

MASTERING
GEOMETRY

BOOK 4 Grades 9–10

A Complete Practice Workbook for Foundations, Angles, Triangles,
Congruence, Similarity, Trigonometry, Polygons, Circles, Area, and Volume

Learn it. Practice it. Prove it.

ANGLES • TRIANGLES • PROOF • TRIGONOMETRY • CIRCLES • VOLUME

THIS BOOK BELONGS TO

Name _____

Class / Period _____

Teacher _____

School Year _____

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Foundations of Geometry

? BIG QUESTION

How can you find the exact distance between two places on a map, or the point exactly halfway between them, without ever picking up a ruler?

BY THE END OF THIS UNIT, I CAN

- ☐ name and describe points, lines, planes, segments, and rays.
- ☐ find the distance between two points on a number line or the coordinate plane.
- ☐ find the midpoint of a segment and use it to find a missing endpoint.
- ☐ copy a segment and construct a perpendicular bisector with a compass and straightedge.

UNIT ROADMAP

1.1 The Building Blocks p. 13

1.2 Segments, Rays, and Distance p. 14

1.3 Midpoints and Basic Constructions p. 16

Suggested time: 1 week

Standards: G-CO.A.1; G-CO.D.12; G-GPE.B.6; 8.G.B.8

KEY VOCABULARY

point

line

plane

segment

ray

collinear

coplanar

midpoint

distance formula

construction

The Building Blocks

KEY TERMS point • line • plane • collinear • coplanar • intersection

Geometry begins with three **undefined terms**: point, line, and plane. They are called undefined because we describe them instead of defining them with other words. Every other figure in geometry is built from these three.

Term	Description	How to name it
Point	A location with no size, drawn as a dot	A capital letter, such as point P
Line	A straight path of points that extends without end in both directions	Two points on it, such as line AB, or one lowercase letter, such as line m
Plane	A flat surface that extends without end in all directions	Three points on it that are not on one line, such as plane XYZ, or one capital script letter

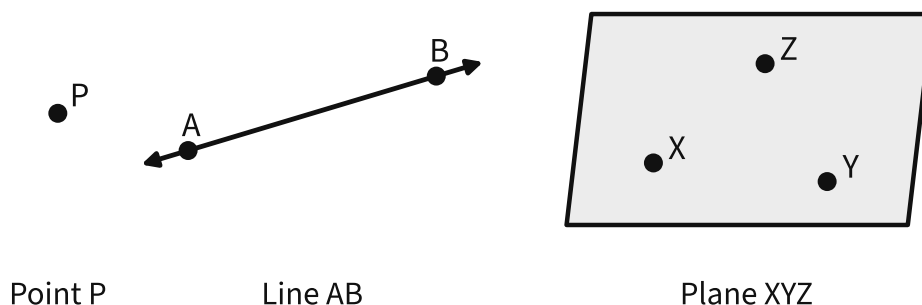


FIGURE 1.1 Point P, line AB, and plane XYZ. Line AB continues without end in both directions, shown by the arrowheads.

Points that lie on the same line are **collinear**. Points that lie in the same plane are **coplanar**. The **intersection** of two figures is the set of points they share. Two lines intersect in a point, and two planes intersect in a line.

✓ QUICK CHECK

- 1 A location with no size is called a _____.
- 2 A flat surface that extends without end in all directions is called a _____.
- 3 Three or more points that lie on the same line are _____.

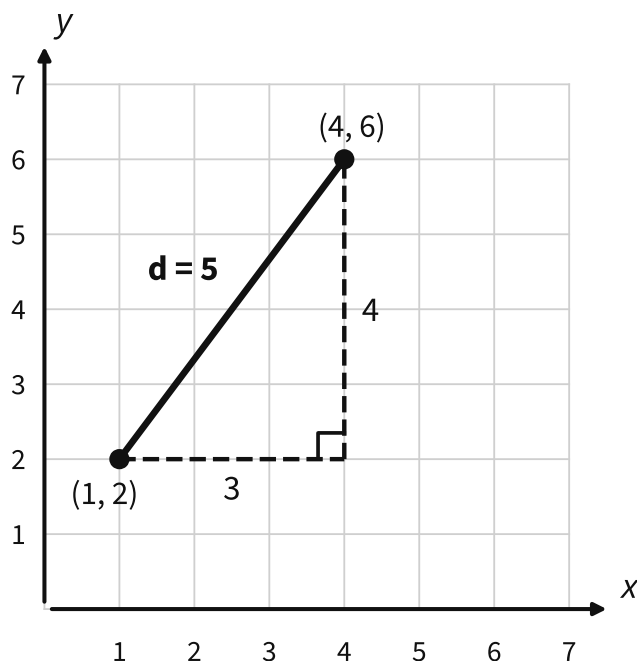


FIGURE 1.3 The distance from (1, 2) to (4, 6) is the hypotenuse of a right triangle with legs 3 and 4.

EXAMPLE 1 Find the distance between (1, 2) and (4, 6)

STEP 1 Find the changes: $x_2 - x_1 = 4 - 1 = 3$ and $y_2 - y_1 = 6 - 2 = 4$.

STEP 2 Substitute: $d = \sqrt{3^2 + 4^2} = \sqrt{9 + 16}$.

STEP 3 Simplify: $d = \sqrt{25} = 5$.

ANSWER 5 units

! SQUARE BEFORE YOU ADD

The distance is not the sum of the horizontal and vertical changes. In Example 1, $3 + 4 = 7$ is the length of a path along the grid lines, but the straight-line distance is 5.

SET 2: MIDPOINT

Find the midpoint of the segment with the given endpoints.

- 1 (2, 4) and (8, 12)
- 2 (-3, 5) and (7, 1)
- 3 (0, 0) and (10, 6)
- 4 (-4, -2) and (4, 6)
- 5 (5, 9) and (-5, -9)
- 6 (1, -7) and (9, 3)

DIAGRAM-BASED PRACTICE

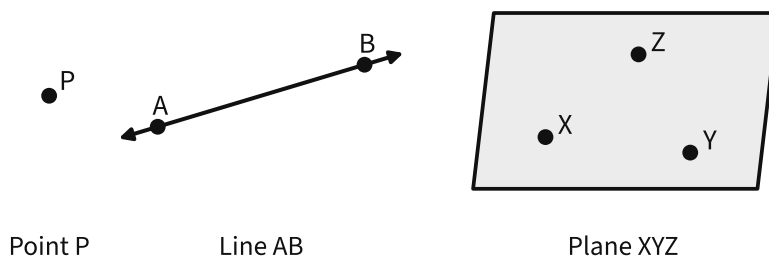


FIGURE 1.6 Use this figure for problems 1 to 4.

- 1 Name two points on line AB.

- 2 Name three points that are coplanar but not collinear.

- 3 Is point P collinear with A and B? Explain.

- 4 Are X, Y, and Z collinear? Are they coplanar? Explain.

Error Analysis

Common Mistake	Do This Instead
MISTAKE Adding the horizontal and vertical changes: the distance from (2, 1) to (5, 5) is $3 + 4 = 7$.	FIX Square each change, add, then take the square root: $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$.
MISTAKE Subtracting the coordinates in the midpoint formula.	FIX Add, then divide by 2. The midpoint of (-2, 6) and (4, 2) is $(\frac{-2+4}{2}, \frac{6+2}{2}) = (1, 4)$.
MISTAKE Writing a squared difference as negative: $(-1 - 5)^2 = -36$.	FIX A squared number is never negative: $(-6)^2 = (-6)(-6) = 36$.
MISTAKE Mixing coordinates, as in $(x_2 - y_1)$.	FIX Label the points (x_1, y_1) and (x_2, y_2) first. Subtract x from x and y from y.
MISTAKE Finding a missing endpoint by averaging the midpoint and the known endpoint.	FIX The midpoint is halfway. Move the same distance again: if A = (0, 1) and M = (2, 4), then B = (4, 7).

Each box shows a student's incorrect work. Describe the error in words, then write the correct solution.

! STUDENT WORK Find the distance between (1, 5) and (4, 9)

STEP 1 $d = (4 - 1) + (9 - 5)$

STEP 2 $d = 3 + 4 = 7$

ANSWER 7

1 What did the student do wrong? Show the correct work.

Unit 1 Review and Challenge

VOCABULARY

1 Write the letter of the definition that matches each term.

1. ____	Point	A. Part of a line with one endpoint that extends without end in one direction
2. ____	Plane	B. Points that lie on the same line
3. ____	Segment	C. A location with no size
4. ____	Ray	D. A drawing made with only a compass and straightedge
5. ____	Collinear	E. The point that divides a segment into two congruent segments
6. ____	Coplanar	F. A flat surface that extends without end in all directions
7. ____	Midpoint	G. Part of a line with two endpoints
8. ____	Construction	H. Points that lie in the same plane

IDENTIFICATION

- 1 A part of a line that has two endpoints is called a _____.
- 2 Three points that lie on the same line are called _____.
- 3 A precise drawing made only with a compass and straightedge is a _____.

CALCULATIONS

- 1 Find the distance between $(3, -2)$ and $(-5, 4)$.
- 2 Find the midpoint of $(8, -6)$ and $(-4, 10)$.
- 3 M is the midpoint of AB, with $A = (4, 1)$ and $B = (-6, 9)$. Find M.
- 4 M is the midpoint of AB. $AM = 4x + 2$ and $MB = 6x - 4$. Find x and AB.

Unit 1 Quiz

Time goal: 25 minutes. 10 questions. Show your work for full credit.

- 1 A flat surface that extends without end in all directions is called a:
☐ (A) point ☐ (B) line ☐ (C) plane ☐ (D) ray
- 2 The distance between $(0, 0)$ and $(5, 12)$ is:
☐ (A) 17 ☐ (B) 13 ☐ (C) $\sqrt{17}$ ☐ (D) 60
- 3 The midpoint of $(4, 8)$ and $(-2, 4)$ is:
☐ (A) $(2, 6)$ ☐ (B) $(3, 4)$ ☐ (C) $(1, 12)$ ☐ (D) $(1, 6)$
- 4 Two rays that share an endpoint and form a line are:
☐ (A) opposite rays ☐ (B) parallel rays
☐ (C) perpendicular rays ☐ (D) skew rays
- 5 Points that lie in the same plane are:
☐ (A) collinear ☐ (B) coplanar ☐ (C) concurrent ☐ (D) congruent
- 6 Find the distance and the midpoint between $(-6, 2)$ and $(4, 14)$.
- 7 Segment AB has midpoint $M(3, -1)$. If $A = (-5, 7)$, find B.
- 8 Two points on a number line are at -12 and 8 . A third point divides the segment in the ratio $1 : 3$, counting from the left endpoint. Find its coordinate.
- 9 On a city map ($1 \text{ unit} = 1 \text{ mile}$), the post office is at $(5, 1)$ and the school is at $(-3, 7)$. How far apart are they in a straight line?

✓ **QUICK CHECK**

- 1 Two lines intersect as in the figure, and $m\angle 1 = 50^\circ$. Find $m\angle 2$, $m\angle 3$, and $m\angle 4$.
- 2 Two vertical angles measure $(5x)^\circ$ and $(2x + 30)^\circ$. Find x .

UNIT 2 | LESSON 2.4

Parallel Lines and Transversals

KEY TERMS parallel lines • transversal • corresponding • alternate interior • alternate exterior • same-side interior

A **transversal** is a line that crosses two or more lines at different points. When a transversal crosses two lines, it forms eight angles. In this book they are numbered clockwise from the upper left at each intersection, as in the figure.

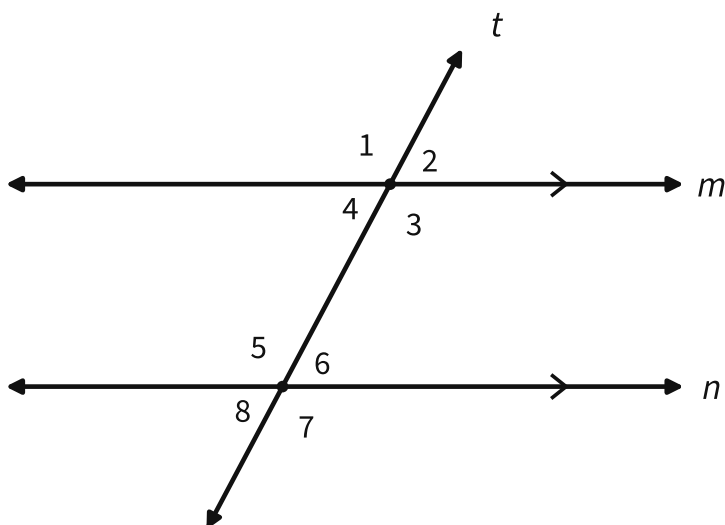


FIGURE 2.5 Parallel lines m and n cut by transversal t . This numbering is used for every parallel-lines problem in the book. Figures are not drawn to scale.

**SET 2: APPLY CPCTC**

- 1 $\triangle ABC \cong \triangle DEF$. $AB = 7$ cm, $AC = 5$ cm, and $m\angle B = 60^\circ$. Find DE , DF , and $m\angle E$.
- 2 $\triangle PQR \cong \triangle STU$. $PQ = 12$, $QR = 9$, and $m\angle P = 35^\circ$. Find ST and $m\angle S$.
- 3 $\triangle XYZ \cong \triangle LMN$. $XZ = 14$, $m\angle X = 50^\circ$, and $m\angle Y = 70^\circ$. Find LN and $m\angle N$.
- 4 $\triangle ABC \cong \triangle DEF$. $AB = 3x + 5$ and $DE = 5x - 11$. Find x and AB .
- 5 $\triangle PQR \cong \triangle STU$. $m\angle P = (4x)^\circ$ and $m\angle S = (2x + 30)^\circ$. Find x .
- 6 $\triangle XYZ \cong \triangle ABC$. $XY = 2x + 4$ and $AB = 3x - 1$. Find x .
- 7 $\triangle GHI \cong \triangle JKL$. $m\angle G = (6x - 8)^\circ$ and $m\angle J = (4x + 12)^\circ$. Find x .

**SET 3: TWO-COLUMN PROOFS**

- 1 Given: $AB \cong DE$, $BC \cong EF$, and $AC \cong DF$. Prove: $\triangle ABC \cong \triangle DEF$. Fill in the missing reasons.

Statement	Reason
1. $AB \cong DE$	1. Given
2. $BC \cong EF$	2. _____
3. $AC \cong DF$	3. _____
4. $\triangle ABC \cong \triangle DEF$	4. _____

- 7 An isosceles triangle has a vertex angle of 44° . Each base angle measures:
- (A) 44° (B) 68° (C) 72° (D) 136°
- 8 Which is NOT a valid congruence method in general?
- (A) SSS (B) SAS (C) ASA (D) SSA
- 9 If $\triangle ABC \cong \triangle DEF$, vertex B corresponds to vertex:
- (A) E (B) D (C) F (D) A
- 10 Two triangles with two pairs of congruent angles are similar by:
- (A) SSS (B) HL (C) AA (D) SAS
- 11 A scale factor of 2 multiplies area by:
- (A) 2 (B) 6 (C) 8 (D) 4
- 12 Solve $\frac{x}{15} = \frac{4}{6}$.
- (A) 8 (B) 10 (C) 12 (D) 22.5
- 13 Which statement about vertical angles is true?
- (A) They are complementary. (B) They require parallel lines.
(C) They are congruent. (D) They add to 90° .
- 14 A triangle with angles 30° , 60° , and 90° is:
- (A) right (B) acute (C) obtuse (D) equiangular
- 15 Two sides of a triangle are 4 and 11. The third side x must satisfy:
- (A) $x < 15$ (B) $4 < x < 11$ (C) $0 < x < 15$ (D) $7 < x < 15$
- 16 Two triangles with all three pairs of corresponding sides congruent are congruent by:
- (A) SAS (B) SSS (C) ASA (D) AAS
- 17 CPCTC is used:
- (A) after triangles are proved congruent
(B) to prove AAA congruence
(C) only with similar triangles
(D) to find polygon angle sums

! STUDENT WORK Find the area of a sector with $r = 6$ and $\theta = 90^\circ$.

STEP 1 $A = \frac{90}{180} \times \pi \times 36$

ANSWER $A = 18\pi$

- 3** What did the student do wrong? Correct the work.

APPLY IT

Real-World Applications

Round to two decimal places and include units.

- 1 Pizza.** A pizza has a 16-in diameter. (a) Find its area. (b) It is cut into 8 equal slices. Find the area of one slice and the length of crust on one slice.

Answer: _____

- 2 Bicycle.** A bike wheel has a 26-in diameter. (a) How far does the bike travel in one turn of the wheel? (b) In 200 turns?

Answer: _____

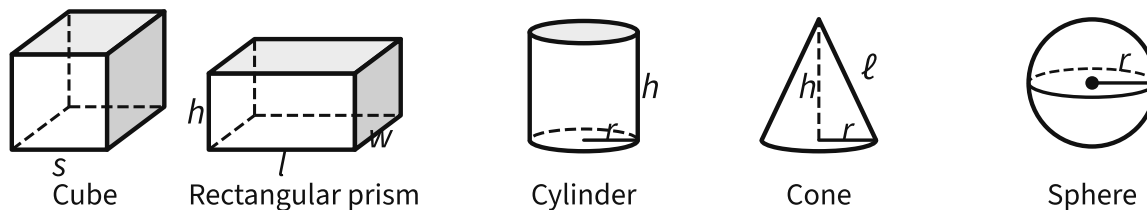


FIGURE 9.3 Common solids. The slant height ℓ of a cone is measured along its side, not straight up.

Solid	Surface area
Cube (edge s)	$SA = 6s^2$
Rectangular prism (l , w , h)	$SA = 2(lw + lh + wh)$
Cylinder (r , h)	$SA = 2\pi r^2 + 2\pi rh$ (two bases plus the lateral surface)
Cone (r , slant height ℓ)	$SA = \pi r^2 + \pi r\ell$
Sphere (r)	$SA = 4\pi r^2$

EXAMPLE 5 Find the surface area of a rectangular prism that is $4 \times 5 \times 6$.

STEP 1 $SA = 2(4 \times 5 + 4 \times 6 + 5 \times 6)$.

STEP 2 $SA = 2(20 + 24 + 30) = 2(74) = 148$.

ANSWER 148 square units

EXAMPLE 6 Find the surface area of a cylinder with $r = 3$ and $h = 10$.

STEP 1 Bases: $2\pi(3)^2 = 18\pi$.

STEP 2 Lateral surface: $2\pi(3)(10) = 60\pi$.

STEP 3 $SA = 18\pi + 60\pi = 78\pi \approx 245.04$.

ANSWER $78\pi \approx 245.04$ square units

30 A right triangle has hypotenuse 25 and one leg 7. Find the angle opposite the leg of length 7. (2 pts)

31 Lines $m \parallel n$. Corresponding angles $\angle 1$ and $\angle 5$ measure $(5x + 10)^\circ$ and $(7x - 20)^\circ$. Find x and $m\angle 1$. (2 pts)

32 $\triangle ABC \sim \triangle DEF$ with $AB = 5$, $BC = 7$, and $DE = 10$. Find EF . (2 pts)

33 Find the total surface area of a cylinder with $r = 5$ and $h = 8$. (2 pts)

34 Find the volume of a cone with $r = 5$ and $h = 12$. (2 pts)

35 Each interior angle of a regular polygon is 150° . Find n . (2 pts)

Section C: Diagram-Based (15 points, 3 points each)

36 Classify each angle as acute, right, obtuse, or straight: 45° , 90° , 130° , 180° . Then sketch one of each and label its measure. (3 pts)

5. Sides 8, $\sqrt{52}$, and $\sqrt{52}$; $P = 8 + 2\sqrt{52} = 8 + 4\sqrt{13} \approx 22.42$; $A = \frac{1}{2}(8)(6) = 24$ square units

CHALLENGE PROBLEMS

- $6x + 30 = 180$, so $x = 25$; the angles are 40° , 40° , and 100° . It is obtuse, and it is isosceles because two angles are congruent.
- The third side is either 5 or 12. Sides 5, 5, 12 fail the Triangle Inequality ($5 + 5 < 12$), so the sides must be 5, 12, 12 and $P = 29$.

Unit 3 Quiz (p. 69)

- B.** Two sides are congruent.
- C.** 180°
- D.** $180^\circ - 130^\circ = 50^\circ$
- A.** $4 + 7 = 11$, which is less than 12.
- C.** $10 - 6 < x < 10 + 6$
- A.** $\frac{1}{2}(10)(7) = 35$
- D.** $3 \times 6 = 18$
- $12x = 180$, so $x = 15$; 45° , 60° , 75° ; acute and scalene
- $3 < x < 19$
- hypotenuse $\sqrt{400 + 441} = 29$; $P = 70$; $A = 210$ square units
- $9 + 10 = 19 > 17$, so it is possible. $s = 18$; $A = \sqrt{18 \times 9 \times 8 \times 1} = \sqrt{1296} = 36$ square units
- 270 m^2
- $2(3x + 5) + (4x + 10) = 180$, so $10x + 20 = 180$ and $x = 16$; base angles 53° , vertex angle 74°

Unit 4: Congruence and Proof Basics

Lesson 4.1 Congruent Figures (p. 72)

- $\angle X$, $\angle Y$, XY , YZ
- ST ; $\angle U$
- PR

Lesson 4.2 Triangle Congruence Postulates (p. 73)

- SSS
- SAS
- ASA

Lesson 4.3 Identifying the Congruence Method (p. 74)

- AAS
- not enough information (SSA)

Lesson 4.4 Introduction to Two-Column Proofs (p. 75)

- Reflexive Property; Reflexive Property; Vertical Angles Theorem

Unit 4 Practice (p. 78)

GUIDED PRACTICE

- $\angle D$

SET 1: IDENTIFY THE METHOD

- SSS
- SAS
- ASA
- AAS
- not enough information (SSA)
- not enough information (AAA proves similarity only)
- not enough information
- HL
- SSS
- SAS
- ASA

SET 2: APPLY CPCTC

- $DE = 7 \text{ cm}$; $DF = 5 \text{ cm}$; $m\angle E = 60^\circ$
- $ST = 12$; $m\angle S = 35^\circ$
- $LN = 14$; $m\angle N = m\angle Z = 180^\circ - 50^\circ - 70^\circ = 60^\circ$
- $3x + 5 = 5x - 11$, so $x = 8$ and $AB = 29$.
- $4x = 2x + 30$, so $x = 15$ (each angle is 60°).
- $2x + 4 = 3x - 1$, so $x = 5$.
- $6x - 8 = 4x + 12$, so $x = 10$.

SET 3: TWO-COLUMN PROOFS

2. Given; 3. Given; 4. SSS
1. M is the midpoint of AB (Given). 2. $AM \cong BM$ (Definition of midpoint). 3. $PM \cong QM$ (Given). 4. $\angle AMP \cong \angle BMQ$ (Vertical Angles Theorem). 5. $\triangle APM \cong \triangle BQM$ (SAS).
1. $AB \parallel CD$ (Given). 2. $\angle ABC \cong \angle DCB$ (Alternate interior angles are congruent). 3. $AB \cong DC$ (Given). 4. $BC \cong CB$ (Reflexive Property). 5. $\triangle ABC \cong \triangle DCB$ (SAS).

DIAGRAM-BASED PRACTICE

- $AB \cong DE$, $BC \cong EF$, $AC \cong DF$, $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$
- Answers vary. $\angle B \cong \angle E$ (ASA); $AC \cong DF$ (SAS); or $\angle C \cong \angle F$ (AAS).
- 60° ($m\angle F = m\angle C = 180^\circ - 50^\circ - 70^\circ$)
- 24

MIXED PRACTICE

- SAS
- 19